Quantum Counterpart Theory

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The project, broadly construed

What is the best interpretation of quantum mechanics for “indistinguishable” systems, in which particles are properly taken as the theory’s subject matter?

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Why should CT need QM?

Isn’t there already a (topic-neutral) argument for Counterpart Theory?

Lewis’s (1986, 2009) argument against trans-world overlap ("endurance through worlds") – the argument from accidental intrinsics – assumes Genuine Modal Realism; ersatzers escape. Quantum considerations address realists and ersatzers alike.
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Constraints on quantum interpretations

I will look only for a “realist” interpretation: treating $\Psi$ as “ontic” rather than “epistemic”.

I will assume that $\Psi$ is complete—at least for the micro-world (so I will not consider e.g. modal theories or de Broglie-Bohm).
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1. Against factorism
   - Factorism defined
     - Factorism is not haecceitism
     - Why factorism is wrong

2. Qualitative individuation
   - Natural decompositions
   - Qualitative individuation

3. Micro-Everettianism
   - Branch-bound particles
   - Endurantism or perdurantism?

4. Problems for Micro-Everettianism
   - A preferred basis problem
   - Possible escapes?
Factorism defined (1)

- Consider an assembly of two “distinguishable” quantum systems. (This means the systems have state-independent properties by which they are absolutely discernible.)
- Let the first system’s Hilbert space be $\mathcal{H}_1$, and the second’s $\mathcal{H}_2$. 
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- So, e.g., $|\xi\rangle_1 \otimes |\eta\rangle_2$ is interpreted as: system 1 being in the state (represented by) $|\xi\rangle$ and system 2 being in the state (represented by) $|\eta\rangle$.
- Any non-separable state counts as entangled.
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Factorism is not haecceitism

Two natural definitions of haecceitism for QM:

1. (Metaphysical.) The acceptance of haecceitic differences (cf. Lewis (1986, p. 221)); i.e., two states may differ solely as to how the systems are embedded in the web of properties and relations.
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Why factorism is wrong (1)

- Under factorism, single-system states may be calculated using the partial trace operation.

\[ \forall |\psi\rangle \in S_\lambda (\mathcal{H} \otimes \mathcal{H}) : \quad \rho_1 := \text{Tr}_2 (|\psi\rangle \langle \psi|) , \quad \rho_2 := \text{Tr}_1 (|\psi\rangle \langle \psi|) \quad (2) \]
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- But for all $|\psi\rangle \in S_\lambda(\mathcal{H} \otimes \mathcal{H})$, $U(\pi)|\psi\rangle\langle\psi|U^\dagger(\pi) = |\psi\rangle\langle\psi|$, so $\rho_1 = \rho_2$. 
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Natural decompositions (1)

- Zanardi (2001) and Zanardi, Lidar & Lloyd (2004): Let $\mathcal{A}$ be the joint algebra we wish to decompose.
- Then we seek two subalgebras $\mathcal{A}_1, \mathcal{A}_2$ of $\mathcal{A}$, satisfying:

  1. Local accessibility. (Sensible quantities.)
  2. Subsystem independence: the subalgebras commute: $\forall A \in \mathcal{A}_1, \forall B \in \mathcal{A}_2: [A, B] = 0$. (3)
     I.e. each system possesses its properties independently of the other.
  3. Completeness: the minimal algebra containing $\mathcal{A}_1$ and $\mathcal{A}_2$ amounts to $\mathcal{A}$, and we have an isomorphism: $\mathcal{A} := \mathcal{A}_1 \lor \mathcal{A}_2 \cong a_1 \otimes a_2$ (4)
     for two "single-system" algebras $a_1, a_2$ such that $a_1 \otimes 1 \cong \mathcal{A}_1$ and $1 \otimes a_2 \cong \mathcal{A}_2$.
     I.e. the assembly has been decomposed without residue.
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  2. **Subsystem independence:** The subalgebras commute:

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(3)

I.e. each system possesses its properties **independently** of the other.

3. **Completeness:** the minimal algebra containing $\mathcal{A}_1$ and $\mathcal{A}_2$ amount to $\mathcal{A}$, and we have an isomorphism:

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for two “single-system” algebras $a_1, a_2$ such that $a_1 \otimes 1 \cong \mathcal{A}_1$ and $1 \otimes a_2 \cong \mathcal{A}_2$.

I.e. the assembly has been **decomposed without residue.**
Natural decompositions (1)

- Zanardi (2001) and Zanardi, Lidar & Lloyd (2004):
  Let $\mathcal{A}$ be the joint algebra we wish to decompose.

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So let us try to apply the Zanardi recipe to $A = B(S_\lambda(H \otimes H))$.

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- So let us try to apply the Zanardi recipe to $\mathcal{A} = \mathcal{B}(S_\lambda(\mathcal{H} \otimes \mathcal{H}))$.
- But dim($S_\lambda(\mathcal{H} \otimes \mathcal{H})$) might be prime!
- So instead I propose to seek natural decompositions of subspaces of the joint Hilbert space $S_\lambda(\mathcal{H} \otimes \mathcal{H})$: 

$$S_\lambda(\mathcal{H} \otimes \mathcal{H}) = \bigoplus_i S_i,$$

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   - Factorism defined
   - Factorism is not haecceitism
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   - Natural decompositions
   - Qualitative individuation

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   - Endurantism or perdurantism?

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   - A preferred basis problem
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Qualitative Individuation (1)

- ‘Qualitative individuation’ means to pick out by appeal to qualitative properties and relations. The entities that are picked out are those that possess the specified properties and relations.

- I assume that the possession of a property or properties by a constituent system is represented by a projector on the single-system Hilbert space $\mathcal{H}$. It need not be one-dimensional ($= a$ maximally specific property).
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I assume that the possession of a property or properties by a constituent system is represented by a projector on the single-system Hilbert space \( \mathcal{H} \). *It need not be one-dimensional* (= a maximally specific property).

So let \( E_{\alpha}, E_{\beta} \) be two individuation criteria, one for each constituent system. *We must have* \( E_{\alpha} \perp E_{\beta} \); i.e. \( E_{\alpha} E_{\beta} = E_{\beta} E_{\alpha} = 0 \).
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Qualitative individuation (2)

Each square represents a product state.

$H$

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Qualitative individuation (2)

Blue squares represent antisymmetrized states

Red squares represent symmetrized states
We now seek two single-system algebras $a_\alpha, a_\beta$ and an isomorphism $\pi_\lambda : A \rightarrow a_\alpha \otimes a_\beta$ such that $A_\alpha := \pi_\lambda^{-1}[a_\alpha \otimes \mathbf{1}]$ and $A_\beta := \pi_\lambda^{-1}[\mathbf{1} \otimes a_\beta]$ satisfy Zanardi et al's three conditions.

Consider $a_\alpha$.

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- $a_\alpha \subseteq B(H)$, since it is a single-system algebra.
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- This narrows down the operators to
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  \{ A \in B(H) \mid A = E_\alpha AE_\alpha \oplus (1 - E_\alpha)A(1 - E_\alpha) \}.
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  But the second component makes no difference for the $\alpha$-system.
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**Micro-Everettianism: Finding the constituents**

**Credo:** Give the same interpretation to each individuation block that the factorist gives to its isomorphic cousin.

- The individuation block \((E_\alpha, E_\beta)\) is the subspace of states in which system \(\alpha\) (the particle individuated by \(E_\alpha\)) and system \(\beta\) co-exist.

- Take any state which maps to a product state under \(\pi\lambda\). It will be of the form \(\frac{1}{\sqrt{2}}(|\xi\rangle \otimes |\eta\rangle \pm |\eta\rangle \otimes |\xi\rangle)\). (7)

  This state is not entangled according to a recent heterodoxy – Ghirardi, Marinatto & Weber (2002).

  Call such states branches.

- Under factorism, an assembly’s state is non-entangled iff its constituent systems occupy pure states.

- Under micro-Everettianism, the assembly is non-GMW-entangled iff 1D individuation criteria suffice to individuate the constituent systems.
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Identity conditions and branch-bound particles

We can use $\mathcal{E}$ as a **trans-branch** (a fortiori trans-state) **identity condition** for particles $\alpha$ and $\beta$:

'has a state in $\text{ran}(E_\alpha)$ and co-exists with a particle in a state in $\text{ran}(E_\beta)$';

Cf. Leibniz's monads; Lewis's world-bound individuals.
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   - A preferred basis problem
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Endurantism or perdurantism? (1)

**Micro-Everettianism** interprets any state of the assembly as the superposition of (non-entangled) branches, consisting of correlated branch-bound particles.

By selecting a branch-bound particle from each branch, we can build arbitrary trans-branch fusions. Such fusions “perdure” through branches.
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Adam Caulton

Quantum Counterpart Theory
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